Cournot competition with 3 firms

Assume a market with 3 firms where companies compete in quantities (Cournot). The inverse demand has the following form: $P = a - bQ$, and the cost for the firms is: $C = qc$.

1. What should be the individual quantities, aggregate quantity, price, and equilibrium profits in this market?
2. Suppose one of the firms acts as if there are 4 other firms in the market while the other firms continue to act as in the previous case. Calculate the individual quantities, aggregate quantity, price, and profits.
3. Now assume one of the firms continues to act as if there are 4 other firms, and the other 2 firms react optimally to the particular firm that acts as if there are 4 other firms. Assume these two firms end up producing the same amount. Calculate the individual quantities, aggregate quantity, price, and profits.
4. In which situation is the price lower? Assume $c = b = 1$ and $a = 2$.

Solution

1. We solve for firm 1:

$$\pi_1 = Pq_1 - cq_1 = (a - bQ)q_1 - cq_1 = (a - bq_1 - bq_2 - bq_3)q_1 - cq_1$$

$$\pi'_{q_1} = a - 2bq_1 - bq_2 - bq_3 - c = 0$$

$$\frac{a - bq_2 - bq_3 - c}{2b} = q_1$$

This holds for the other firms, so $q_1 = q_2 = q_3$.

$$\frac{a - bq_1 - bq_1 - c}{2b} = q_1$$

$$\frac{a - c}{2b} - q_1 = q_1$$

$$q_1 = \frac{a - c}{4b}$$

Therefore, $q_2 = q_3 = \frac{a - c}{4b}$. Calculating the price:

$$P = a - bQ = a - b \cdot 3\frac{a - c}{4b} = a - 3\frac{a - c}{4}$$

$$P = \frac{a + 3c}{4}$$

Calculating the profit for each firm:

$$\pi = \frac{a + 3c}{4}\frac{a - c}{4b} - c\frac{a - c}{4b}$$

$$\pi = (a + 3c)\frac{a - c}{16b} - 4c\frac{a - c}{16b} = \frac{a^2 - ac + 3ca - 3c^2 - 4ca + 4c^2}{16b}$$

$$\pi = \frac{a^2 - 2ac + c^2}{16b} = \frac{(a - c)^2}{16b}$$

2. Introducing the 4 firms into the best response function of the firm:

$$\frac{a - bq_1 - bq_2 - bq_3 - bq_4 - c}{2b} = q$$

Assuming that the 4 firms produce the same amount:

$$\frac{a - bq - bq - bq - bq - c}{2b} = q$$

$$\frac{a - 4bq - c}{2b} = q$$

$$\frac{a - c}{2b} - 2q = q$$

$$\frac{a - c}{2b} = 3q$$

$$q = \frac{a - c}{6b}$$

Calculating the aggregate quantity, assuming the other firms continue producing the same amount:

$$q_1 = q_2 = \frac{a - c}{4b}$$

$$Q = 2\frac{a-c}{4b} + \frac{a-c}{6b} = \frac{a-c}{2b} + \frac{a-c}{6b} = \frac{2(a-c)}{3b}$$

$$P = a - bQ = a - 2\frac{a-c}{3} = \frac{3a-2a+2c}{3}$$

$$P = \frac{a+2c}{3}$$

The profits for the particular firm:

$$\pi = \frac{a+2c}{3}\frac{a-c}{6b} - c\frac{a-c}{6b} = \frac{a^2 - ac + 2ca - 2c^2 - 3ca + 3c^2}{18b}$$

$$\pi = \frac{a^2 - 2ac + c^2}{18b} = \frac{(a-c)^2}{18b}$$

The profits for the other firms:

$$\pi_1 = \pi_2 = \frac{a+2c}{3}\frac{a-c}{4b} - c\frac{a-c}{4b} = \frac{a^2 - ac + 2ca - 2c^2 - 3ac + 3c^2}{12b}$$

$$\pi_1 = \pi_2 = \frac{a^2 - 2ac + c^2}{12b} = \frac{(a-c)^2}{12b}$$

3. Introducing $\frac{a-c}{6b}$ into the two firms:

$$q = \frac{a - bq - b\frac{a-c}{6b} - c}{2b}$$

$$q = \frac{a - bq - \frac{a-c}{6} - c}{2b}$$

$$q = \frac{a - bq - \frac{a-c}{6} - c}{2b}$$

$$q = \frac{a - bq - \frac{a-c}{6} - c}{2b}$$

$$q = \frac{-q}{2} + 5\frac{a-c}{12b}$$

$$\frac{3}{2}q = 5\frac{a-c}{12b}$$

$$q = 5\frac{a-c}{18b}$$

Calculating the price:

$$P = a - bQ = a - b\left(5\frac{a-c}{18b} + 5\frac{a-c}{18b} + \frac{a-c}{6b}\right) = a - \left(10\frac{a-c}{18} + \frac{a-c}{6}\right)$$

$$P = \frac{18a - 10a + 10c - 3a + 3c}{18} = \frac{5a + 13c}{18}$$

Calculating the profits of the two firms:

$$\pi_1 = \pi_2 = 5\frac{a-c}{18b}\frac{5a+13c}{18} - 5c\frac{a-c}{18b}$$

$$\pi_1 = \pi_2 = 5\frac{a-c}{324b}\frac{5a+13c}{1} - 5c\frac{a-c}{18b} = \frac{(5a-5c)(5a+13c)-90ca+90c^2}{324b}$$

$$\pi_1 = \pi_2 = \frac{25a^2+65ac-25ca-65c^2-90ca+90c^2}{324b} = \frac{25a^2-50ac+25c^2}{324b}$$

$$\pi_1 = \pi_2 = \frac{25(a-c)^2}{324b}$$

Calculating the profit of the particular firm:

$$\pi = \frac{5a+13c}{18}\frac{a-c}{6b} - c\frac{a-c}{6b} = \frac{5a^2-5ac+13ca-13c^2-ca18+18c^2}{108b}$$

$$\pi = \frac{5a^2-10ac+c^2}{108b} = \frac{5(a-c)^2}{108b}$$

4. Summarizing the results:

Profit (π)	Price (P)	Individual Quantity (q_i)	(Q)
$\frac{(a-c)^2}{16b}$	$\frac{(a+3c)}{4}$	$\frac{(a-c)}{4b}$	$\frac{3(a-c)}{4b}$
$\pi = \frac{(a-c)^2}{18b}$; $\pi_{otros} = \frac{(a-c)^2}{12b}$	$\frac{(a+2c)}{3}$	$q = \frac{(a-c)}{6b}$; $q_{otros} = \frac{(a-c)}{4b}$	$\frac{2(a-c)}{3b}$
$\pi = \frac{5(a-c)^2}{108b}$; $\pi_{otros} = \frac{25(a-c)^2}{324b}$	$\frac{(5a+13c)}{18}$	$q = \frac{(a-c)}{6b}$; $q_{otros} = \frac{5(a-c)}{18b}$	$\frac{13(a-c)}{18b}$

The prices in the different situations are: 1.25, 1.33, and 1.27. Situation 1 has the lowest price.